1. Introduction to Signals and Systems

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1.1. Introduction to Signals

A signal is a function of one or more independent variables. It can be, for example, a voltage or an image. In our discussion, a signal is considered as a function of time. However, we should realize that the addressed theory may also apply to other cases.

A continuous-time signal, also called an analog signal, is defined along a continuum of time. An example of continuous-time signals is given in figure 1.1.

\[ x(t) = Ae^{at}, \quad t \geq 0 \]

Figure 1.1. A Continuous-Time Signal.
A discrete-time signal is defined at discrete times. Figure 1.2 gives an example of discrete-time signals. A discrete-time signal is called a digital signal if its amplitude is quantized to a series of discrete levels. An example of digital signals is given in figure 1.3.

\[ x(n) = Aa^n, \ n = 0, 1, 2, \ldots \]

Figure 1.2. A Discrete-Time Signal.

\[ x(n) = \text{round}(Aa^n), \ n = 0, 1, 2, \ldots \]

Figure 1.3. A Digital Signal.
1.2. Introduction to Systems

A system is a device which converts an input signal into an output signal. The input signal is also called excitation, and the output signal is also called response.

A continuous-time system is a system whose input and output are continuous-time signals (figure 1.4). A discrete-time system is a system whose input and output are discrete-time signals (figure 1.5).

A continuous-time system may be implemented by a discrete-time system (figure 1.6). A discrete-time system may be implemented by a continuous-time system (figure 1.7).

\[
x(t) \rightarrow y(t) = T[x(t)] \rightarrow y(t)
\]

Figure 1.4. A Continuous-Time System.
Figure 1.5. A Discrete-Time System.

Figure 1.6. Implementation of a Continuous-Time System by a Discrete-Time System.

Figure 1.7. Implementation of a Discrete-Time System by a Continuous-Time System.
1.3. Classification of Systems

1.3.1. Linear Systems versus Nonlinear Systems

A system is linear if it satisfies the following condition: When the input is a weighted sum of several inputs, the output will be the weighted sum of the corresponding outputs. Otherwise, the system is nonlinear.

For Continuous-Time Systems: Let \( x_1(t) \) and \( x_2(t) \) be two arbitrary signals, and \( a_1 \) and \( a_2 \) be two arbitrary constants. System \( y(t)=T[x(t)] \) is linear if and only if

\[
T[a_1 x_1(t)+a_2 x_2(t)]=a_1 T[x_1(t)]+a_2 T[x_2(t)].
\]  

(1.1)

For Discrete-Time Systems: Let \( x_1(n) \) and \( x_2(n) \) be two arbitrary signals, and \( a_1 \) and \( a_2 \) be two arbitrary constants. System \( y(n)=T[x(n)] \) is linear if and only if

\[
T[a_1 x_1(n)+a_2 x_2(n)]=a_1 T[x_1(n)]+a_2 T[x_2(n)].
\]

(1.2)
Example. Determine whether the following systems are linear:

1. \( y(t) = tx(t) \).
2. \( y(t) = x^2(t) \).
3. \( y(n) = \text{Re}[x(n)] \).
4. \( y(n) = 2x(n) + 3 \).

1.3.2. Time-Invariant Systems versus Time-Variant Systems

A system is time invariant if a time shift of the input results in the same time shift of the output. Otherwise, the system is time variant.

For Continuous-Time Systems: Let \( t_0 \) be an arbitrary real number. System \( y(t) = T[x(t)] \) is time invariant if and only if

\[
T[x(t-t_0)] = y(t-t_0).
\]  (1.3)

For Discrete-Time Systems: Let \( n_0 \) be an arbitrary integer. System
y(n)=T[x(n)] is time invariant if and only if
\[ T[x(n-n_0)] = y(n-n_0). \] (1.4)

Example. Determine whether the following systems are time-invariant:

1. \( y(t) = \sin[x(t)] \).
2. \( y(n) = nx(n) \).
3. \( y(t) = x(2t) \).

1.3.3. Memoryless Systems versus Systems with Memory

A system is memoryless if the output at any time depends only on the input at the same time. Otherwise, the system has memory.

Example. Determine whether system \( y(n) = x(n-n_0) \), where \( n_0 \) is an integer, is memoryless.
1.3.4. Causal Systems versus Noncausal Systems

A system is causal if the output at any time depends only on the input at and before this time. Otherwise, the system is noncausal.

A noncausal system cannot be implemented in real time.

Example. Determine whether the following systems are causal:

(1) \( y(n) = x(-n) \).

(2) \( y(t) = x(t) \cos(t+1) \).

1.3.5. Stable Systems versus Unstable Systems

A signal is bounded if there exists a finite constant such that for any time, the absolute value of the signal is less than or equal to the constant. Otherwise, the signal is unbounded.

A system is stable if its every bounded input produces a bounded output. Otherwise, the system is unstable.
Example. Determine whether the following systems are stable:

(1) \( y(t) = tx(t) \).

(2) \( y(t) = e^{x(t)} \).

1.3.6. Invertible Systems versus Noninvertible Systems

System A is invertible if system B exists such that when A and B are cascaded, the output of B is equal to the input of A (figures 1.8 and 1.9). B is referred to as the inverse system of A. Otherwise, A is noninvertible.

There exists a one-to-one correspondence between the input and the output of an invertible system: (1) For a given input, the output can be determined uniquely. Actually, both invertible systems and noninvertible systems satisfy this condition. (2) For a given output, the input can be determined uniquely. Only invertible systems satisfy this condition.
Example. Determine whether the following systems are invertible:

(1) \( y(t) = 2x(t) \).

(2) \( y(n) = \sum_{k=-\infty}^{n} x(k) \).

(3) \( y(n) = 0 \).

(4) \( y(t) = x^2(t) \).